General Schemas Theory and N-Categories

The Advance of the Systems Engineering Discipline through an extension of Systems Theory

Kent D. Palmer, Ph.D.

P.O. Box 1632
Orange CA 92856 USA
714-633-9508
palmer@exo.com

Started 10/27/03; Version 0.04; 11/02/03; gs05a04.doc

Keywords: Systems Engineering, Systems Theory,

Introduction

In this paper I will attempt to explore the implications of N-category Theory and General Schemas theory. Here we begin to enter an area in which I am quite unsure about but which I believe is important and should not be left out. N-Category theory has been introduced by John Baez[1] from University of California, Riverside. But there are other innovators in this area such as Tom Leinster[2] who has reviewed all the different versions of N-category theory and compared them with his own. Since Category Theory itself is hard, then it's generalization to n-categories is even harder. So we will be taking baby steps here and not trying to go too far, but just far enough to show the promise of this area of research with regards to General Schemas Theory.

So let us start by attacking the question Why N-Category theory? Why is that the next step on our journey to try to understand the roots of General Schemas Theory. The answer is this. We have been studying the Pascal Simplicies. They basically give us all possible objects. Category theory itself ignores the objects and focuses on the arrows, or transformations, or maps between objects. When you have the maps you can forget the objects mapped and just use the arrows to describe a particular category. Zero-category theory is basically about the objects. One-category theory concerns the arrows. Two-Category theory concerns the meta-mappings between arrows. Three-Category theory concerns the meta-meta-mappings between the meta-arrows and so forth. It soon gets hard to understand. But the key thing we want to consider here that the mathematicians are not considering is whether it is possible to have negative and imaginary category theory as well as positive category theory. Notice that zero-category theory is not empty but in fact includes all the possible objects generated by the pascal simplicies. Which are all possible objects, period. If an arrow is positive one category theory, then what is a negative one category theory morphism? And beyond that we can ask if there are any such thing as imaginary morphisms. As we go up the series of positive morphisms we get the positive two morphisms called functors, and the positive three morphisms called natural transformations. I don't know if there is a name for positive four morphisms but they would be meta-meta-meta mappings between natural

[1] http://math.ucr.edu/home/baez/README.html

[2] http://www.maths.gla.ac.uk/~tl/index.html#book

transformations. It becomes quickly very hard to figure out what these meta-n-morphisms might mean. Even harder is it to figure out what a negative or an imaginary morphism might mean. Perhaps they mean nothing, but we will take a chance and explore the territory because we think that if there is an answer to this question the answer could be significant for us. This is sort of like trying to figure out if there is a category theory analog to Etter's Link Theory. His link theory basically takes the Pascal Simplicies determination of possible elements and gives each one a count which can be either negative or imaginary as well as zero or positive. Now we can ask the question whether if we pop up out of the zero category will there be any negative or imaginary categories in the opposite direction from the positive categories discovered by n-category theory. In other words will there be a nice symmetry like we get with Etter's link theory that tells us something important about existence that the mathematicians cannot see just like they cannot see negative dimensionality. Perhaps there is nothing to see. But we will never know unless we circle around the area for a while and take a few swipes at trying to define what a negative and an imaginary category might be like. So we need to ruminate on the nature of an arrow, i.e. a morphism. A morphism is a mapping or transformation. It is hard to think of what a negative transformation or a negative mapping might be like. We are talking about a change in time or a change I space when we talk about a negative transformation or a negative mapping. Reversing the arrows is not enough, they are still arrows. All of category theory has the mirror like quality of the Pascal Triangle and other simplicies in as much as when you reverse the arrows for a category you get the anti-category. But as in the case with Set you do not automatically get the real opposite such as Mass. Rather you get a kind of empty opposite like the Anti-Set. So reversing the arrows is not enough. Now this might be related to the Bekenstein Bound which says that a certain amount of information is scrambled not just lost when you drop down from an emergent level. We posit that what ever is not a transformation or a mapping has something to do with emergence, i.e. the difference between the anti-Set and the Mass. If we apply the work of Greimas then we see that the Mass is actually the non-dual between Anti-Set and Non-Set.

One direction we have suggested previously is to move out of determinate Category theory and begin thinking about Arrows as probabilistic, then possibilistic, then propensitistic. The bain of mathematics is that it all takes place in the Present at Hand and it is hardly informed about the different kinds of Being. So we can begin to traverse the meta-levels of Being and build a category theory at each level. But this does not answer the positive question of what is something that is the opposite of a morphism in time or space that contributes to emergence. We could apply the same reasoning to these arrows that we have applied to mass and say that there are anti-arrows and non-arrows and that there is some non-dual-arrow beyond both of them. That non-dual arrow is what lies in the negative and imaginary category theory. But although we have the idea of the Mass as being robustly opposite the Set we do not really have a theory of what lies robustly opposite the morphism.

However, previously we have mentioned that the opposite of the Set/Mass distinction was another distinction taken from physics of the difference between Field/Reserve. Fields deal with invisible lines of force that yield intensities. Reserves deal with some non-measurable quantities which can only be found through accounting and conservation of quantities. They are not just invisible but non-manifest in some sense. But reserves show up in transformations, i.e. morphisms in time, while fields show up by invisible lines in space. So does this not sound somewhat familiar. Perhaps the role of reserves and fields is more significant that we expected previously. What if reserves and fields were not just complementarities of the Set/Mass dualities but were also categorical in the sense that one related to categories in time and the other in space. For instance the reserve of Energy has to be tracked through time by mass/energy accounting schemes. Energy/mass itself is non-manifest and only shows up because we assume that we can do accounting between its various forms to show conservation. In the other hand fields always appear in space as unseen forces with different intensities at different places in space. So these two complementarities of set and mass appear to be categorical in some sense giving us either transformations in time or mappings in space. Time and space are intimately involved here in ways usually ignored by mathematical approaches which are assuming pure presence. We are also assuming that the Reserve is related to Hyper Being and the Field is related to Wild Being just as the Mass is related to Process Being and the Set is related to Pure Being. So as we go back and forth between the elements and the categories we are really making a transition between the different kinds of Being as well within the Multi-lith. But both Fields and Reserves relate to 1-categories because they are either invisible forces that are giving mappings in space or they are non-manifest entities giving transformations in time. However notice the prevalence of talk about invisibles and non-manifest entities. This makes us think that perhaps we are talking about not positive one categories but perhaps negative one categories. Positive one categories might be seen as the positive mappings and positive transformations we posit when we study the fields and reserves. But the nature of fields and reserves themselves may be negative dimensional. It is our projection that is positive, we posit the various mappings and transformations postivisticly but the reality of fields and reserves can be seen as negative in the sense of a withholding of something, for instance in fields forces are invisible, only their results can be seen. On the other hand some things like energymatter are utterly non-manifest in terms of their obeying conservation laws. We posit the conservation and then do the accounting to see whether or not the quantity is conserved or not. The actual thing that connects the various manifestations of matter and energy in situ is something non-manifest beyond our projections of accounting strategies. So we suddenly see a balance here between ontic and ontological properties. The ontic properties out there in the things are what is hidden or non-manifest and it is balanced by the ontological projections which we see in the schemas. It is the schemas that we use to get some insight into what is missing, either as invisible or unmanifest. We use the schemas to see what is visible and manifest and then infer the presence of non-manifest or non-visible entities based on the behavior of what we can see. Slowly we begin to see that this difference between negative and positive categories is a lot like the difference between ontic and ontological which is quite interesting. We discover the peculiarities of the ontic by a kind of negative seeing which takes what is visible and then works out what is either non-visible or non-manifest within the phenomena that is hidden. We then present these results positively as maps and transformations that we give the status of knowledge. This knowledge of the positive maps and transformations is ontological, i.e. part of the projection. But here now the projection is aligned giving hopefully a positive picture of what is hidden or invisible within the ontic.

So perhaps we are really dealing with negative categories all the time and converting them into positive categories that are part of the deterministic projection we call knowledge of the physical world. The positive categories cover over the negative quality of the negative expression of the phenomena as ontic. So science is really looking into a mirror. It looks at the ontic and sees as a reflection the ontological. The negative one categories are really the reserve and field which are related to space and time and are a projection of the higher meta-levels of Being. Over these we project the positive one categories that are our knowledge of how fields and reserves work in physics, but the actual phenomena that we are distinguishing is negative in the sense of being things which are missing but whose actions and shadows we are seeing. Now this would suggest that the deeper we go into the negative categories the more we will build positive projections as we look into the mirror of the ontic and see the visage of the ontological. If we go deeper into the negative two categories we are going to see things like spacetime and matterenergy, i.e. intervals with phase spaces instead of discrete transformations in time or mappings in space. In other words the mappings and transformations will get entangled. But we will see this as transformations between categories or functors.

If when we go into the negative two categories we are getting intervals then the next stage must be intervals between intervals which I call quadratic intervals. I did work on Quadratic Intervals when I was doing the working papers for my previous Dissertation. A Quadratic Interval is a set of four intervals that combine to give a higher order interval like structure. Basically a Quadratic Interval is a tetrahedron of intervals where each side is an interval like structure. Later I discovered that Jung had a similar archetypal structure he called a Quaternity that appears in his Alchemical works like Aion. Physics really does not talk very much about meta-intervals but more or less stops with intervals in its explanation of special and general relativity. So it appears that we could align the functors between categories with the intervals and the natural transformations between functors and the quadratic interval. Each time we go up an n-category level it is harder to think what is at the next level, just like going up the kinds of Being levels. So it is difficult to say what is beyond the quadratic interval and the natural transformation. Certainly there is no name for n-category beyond the natural transformation in mathematics that I know of. But for the moment perhaps this is enough to give us some perspective on the relation between n-categories and the simplicies. The n-categories generalize the simplicies where instead of objects we have a hierarchy of morphisms. There are negative and positive aspects of these morphisms which relate to the ontic and ontological respectively. But then the question arises what about the imaginary morphisms, does the singularity at negative one split as it does for objects giving us imaginary morphisms out of the negative one morphism? In other words, we talk about imaginary numbers as a model of interpenetration of things. But this interpenetration is static. What happens on the level of morphisms when the interpenetration becomes dynamic, in Buddhism borrowing from its Hindu heritage this is called karma, and the relation of karma to the interpenetration is called the tathagata gharba (womb of thus come) as described in The Awakening of Faith sutra. In other words Buddhist metaphysics is the only one I know where this question has been raised as to the imaginary morphisms that split out of the negative one category morphisms and thus produce imaginary morphisms rather than merely static morphisms as those of field and reserve related to invisibles or unmanifests in relation to space and time. In each case one negative morphism remains real and the other one becomes imaginary. And these form a hierarchy related to the quaternions, octonions, sedenions that is different from the progression of intervals into the quadratic interval. This imaginary realm with respect to time has been called by some the *absolute past*. Deleuze for instance talks about it this way. It could be called the mythic time, once upon a time. There are also mythic or imaginary places where the edges of the map curl up and maps used to say that there were monsters toward the edges of the known world. So both in terms of imaginary time and places these imaginary morphisms play an important role. Culturally of course we produce and consume these imaginary worlds like those of JRR Tolkein all the time but we do not give them an ontological standing that takes them seriously in terms of our theorizing or our systematizing. So that is really what we have to consider, whether they play an important role with respect to general schemas theory which we understand as the first refuge of the emergent eventity. There are certain theorists like Jacque Lacan who have raised the imaginary and symbolic onto an equal footing with the real, thus taking it seriously. Also Deleuze and Baudrillard talk about the virtual in a way that takes these imaginary shadows of things very seriously. The question is whether we should take them serious with regards to systems theory and systems engineering.

We have noted that the schemas are the inverse of the dimensionality that falls out of the Pascal Triangle and the Simplicies. We know that the schemas are related to spacetime. They are related to space in as much as they are envelopes that contain things that have emerged. They are related to time by Kant as a way of implementing his categories. We have noted that the simplicies satisfy the conditions of Kant's categories. Strangely they also satisfy the conditions of the non-duals of the Western Worldview. By moving away from the objects produced by the simplicies toward the morphisms and considering both positive, negative and imaginary morphisms we are in fact moving into the realm of the schemas themselves. In fact we can ask whether there is anything beyond the supervenience of the n-categories and the schemas? Because as we see the n-categories when taken fully as being positive, negative and imaginary begins to fill out spacetime as we expect the schemas to do. So this becomes our problem to consider for this essay, is there anything beyond the n-categories in the emergence of the schemas. Our answer will start off as yes. That is because there are aspects to the schemas that cannot be reduced to the ncatergories. But the supervenience of the schemas onto the n-categories and simplicies takes us a long was into understanding the nature of the schemas. We are in fact building up slowly a picture of the infrastructure that gives rise to the schemas especially if we consider the simplicies as objects and n-categories as the morphisms on those objects and broaden our perspective to include negative and imaginary morphisms as well.

This is quite unexpected. Why should n-categories fill in between the simplicies and the schemas another foundational infra-structural level? It makes us wonder if there are other levels to fill in between the simplicies and n-categories that would completely bridge the gap between the schemas and their supervenient infrastructure. Is there something beyond the n-categories? If we think of the application of the meta-levels of Being to the system then we see there that the meta-levels are the language/game system then grammar/rules, then the phonemes/pieces, then the anomalies. So if we consider that the simplicies are like the pieces that appear at the Hyper Being level, and the n-category morphisms are like the grammar/rules that constrain the pieces then there are two things missing. One of the things is the system itself as language/game and the other is the anomalies. The system in this case is what is coming into being: the phenomena considered as a whole. Around that phenomena wraps the n-categories and the simplicies and also some anomalies to produce the schematized emergent event that we experience. From the anomalies the organization of the different schemas as set-like unique organizations arise. Each anomalous schema type organizes the n-categories differently around the dimensional objects that fall under that schema. In this way we can think of the schemas as being produced as the meta-levels of the phenomena itself that accrete as they enter the world. In other words the schemas are part and parcel of the movement of the emergent event into the world. But then we have to consider also the phenomena as a meta-system. The organization of the meta-levels of the meta-system are different from the organization of the system. For the meta-system there is first the complementarities, then the resources then the singularities. This is why we see so many complementarities in the structuring of the schemas. Also they act as a kind of resource distributor and they unfold around singularities which is a completely different type of containment than that of the system. So here we are considering schemas as a whole as having some of the aspects of one or another of the schemas. In this way we finally get a glimpse of what is necessary to start understanding how it could be that we are building up the schemas from out of the simplicies and the n-categories and other elements that form their infra-structure. We can build up layer by layer the necessary components to approximate the emergence of the schemas themselves. In this way we get a glimpse of the infrastructure that allows the supervenience of the schemas over the simplcies and the n-categories. This is so strange that the schemas in fact apply to themselves. That the schemas as a whole have the meta-levels corresponding to the mta-levels of Being and that they as a whole can be both system and meta-system and probably in terms of other schemas as well. The schemas are a unique set of fileters. But as a whole they act as a mass and apply some of the aspects of the various schemas to themselves in a fused way like the Pascal Simplicies. Working out how to loop that loop will take some time and energy. But it is important because the schemas don't just appear from nowhere, they self-produce themselves as a side effect of emergence because emergent phenomena need to be enveloped in the world. The schemas are the way that self-envelopment of phenomena by phenomena in spacetime works.

Multi-Categories and Multi-Spheres

In this section we will use Tom Leinster's Higher Operads, Higher Categories[3] as a reference point for moving in another direction than that which Leinster and Baez have moved in their exploration of n-category theory. Instead we want to consider whether there is a mass-like dual to category theory and multi-category theory, that off-sets its set-like basis. Our argument is going to be somewhat torturous of necessity because we are attempting to push in a fundamentally different direction than the mathematical tradition has gone. In order to do this we will use some aspects of mathematics to guide us in our establishment of the possibility of a dual to category theory and multi-category theory. We are going to call this new theory, for lack of a better word n-blob theory. A blob is a mass-like dual to a category. Just as set particulars are 0-categories then mass instances are 0-blobs. Just as there are multi-categories like those described by Leinster, there are multi-blobs that are the extension of blobs. N-categories are concerned with arrows. If we try to think what is the opposite of an arrow it is something that contains already both domain and co-domain, it is a boundary. The first order category concerns arrows between set elements. We forget the elements and just treat the arrows. So the first order blob concerns boundaries that encompass the mass instances. The second order category concerns the meta-arrows between first order arrows. Of course, arrows are morphisms. The second order blob concerns the meta-boundaries that contain masses. Second order arrows are called functors that operate between categories. So second order boundaries are called tissues in blob theory. Third order arrows between functors are called natural transformations. Third order boundaries encompassing tissues will be called natural bags. Fourth order arrows between natural bags are called modifications. Fourth order boundaries encompassing natural bags are called tweaks.

Tweak	Modification
Bag	Natural Transformation
Tissue	Functor
Boundary of Mass (circle or sphere)	Morphism (arrow)
Mass of instances	Set of particulars

Of course, these names are somewhat humorous. But we are driven to these extremes if we are trying to find words to express this dual of the mathematical n-category theory because there are not really good words that are duals of those chosen by the mathaticians. But the words themselves, like all mathematical words are just chosen more or less at random, and it is the mathematical ideas that are what matter. And here the idea is that there is a dual of n-category or n-multi-category theory which we will call n-blob or n-multi-blob theory until some better terminology shows up.

A key point is that where as category

[3] Cambridge UP 2003

theory with its objects and arrows is a model of transcendence, blob theory is a theory of immanence. We cannot even use G. Spencer-Browns Laws of form or any other sort of boundary logic that assumes that boundaries can be transcended. Blob theory assumes that boundaries cannot be transcended, that you are stuck within what ever boundary you happen to be inside. But also there is the fact that as you go up meta-levels the nature of the boundaries change to that inside and outside become less defined. For instance we can think of meta-blobs as two-spheres and there are ways of getting out of one-spheres in two spheres without crossing any boundaries. There is a homeomorphism between blob theory and n-spheres and other topological constructions such as non-orientable surfaces. Blob theory in a certain sense is like topology except instead of being interested in just the surfaces themselves we are interested in the instances that are enveloped by the surfaces. This enveloping is the way immanence shows up in blob theory that is the opposite of transcendence in category theory. All of Western Philosophy and Science is about Transcendence. This is what produces the duality at the heart of the Western worldivew. All we have to do is think of mind/body, or spirit/matter and other such dualisms to understand that this worldview is totally caught up in dominance relations between duals that are artificially far a part. Thus dualism always generates the intensification of nihilism which is the fundamental ontological phenomena at the heart of the worldview. Only in modern times have philosophers turned to the task of attempting to create immanent philosophies and psychologies that explain how consciousness is embedded in the flesh, like Merleau-Ponty did. It is interesting that the modes of cognition associated with the kinds of Being, which are Pure = pointing, Process = grasping, Hyper = bearing, and Wild = encompassing make the transition between the transcendence of Pure Being to the immanacnce of the Flesh of Wild Being that Merleau-Ponty sought to understand in <u>The Visible and the Invisible</u>. So one way to understand the kindness of Being is as a transition between transcendence and immanence. Category theory as it stand is totally ensconced in Pure Being. We can begin to think of Blob Theory as a being an outpost of Wild Being and as describing the meta-levels of encompassing associated with the mass-like approach to things. Pure Being on the other hand has since Aristotle been associated with the Set-like approach to things. This means that the other two kinds of Being are somehow an interspace between these two extremes of transcendence and immanence. We need to pay particular attention to the significance of the other two kinds of Being from the multilith in their relation to transcendence and immanence. Hyper Being and Process Being set up the interspace that separates transcendence from immanance. Thus our two kinds of fundamental mathematics cannot ignore this difference that makes a difference. Blob Theory must be altered fundamentally by the nature of Wild Being and its distance from Pure Being across the gulf of Hyper and Process Being. However, for now we will merely set up an image of Blob Theory as if it could be represented in Pure Being like Category Theory and Sets in order to explore the resources in Mathematics that we can call on to help us discriminate between these duals.

Now we started out by studying simplicies which are triangles, tetrahedrons, pentahedrons etc defined by the Pascal Triangle. These are all the most basic forms in each dimension. And these forms define the possibility of each dimension in the infinite hierarchy of dimensions. It is B. Fuller who told us that a system must have the shape of a tetrahedron in his book Synergetics, and thus constructed a bridge between geometrical thinking and systems thinking at a fundamental level. But we went on from our description of simplicies to recognize that we must also consider n-categories and n-multicategories in order to have a complete theory of the fundamentals of general schemas theory, our generalization of systems theory to all the possible schemas. Now what I want to propose following B. Fuller's lead, is that there are two other Platonic solids in all dimensions, the so called cross polytope and the hyper cube or tesseract. In the third dimension we know these as the octohedron and the cube. B. Fuller shows that a cube is made of two interpenetrated tetrahedrons while the octohedron is made up of fused or intrapenetrated tetrahedrons. It is a little known fact that the octohedron and the twenty four cell have the special property that if arrows replace the lines between points that these arrows can be so arranged not to interfere with each other. Thus octahedrons and twenty-four cell polytopes in the fourth dimension have this special property that they induce flow without resistance and singularities. We need to keep this special property of octahedrons in mind as we proceed. Now what I want to propose following the lines of thinking first established by B. Fuller is that the cube or tesseract is the model of category theory and n-category theory, and especially n-multicategory theory. In other words all the associative diagrams of category theory are Cartesian in nature. That is to say that they make the most of orthogonalities between things. The interpenetratated tetrahedrons that make up the cube are oriented orthogonally to each other. This is one way for two systems to interact. But there is also the other way for them to interact which is fusion, or intrapenetration. I would like to suggest that blob theory, and n-blob theory and especially n-multiblob theory is related to the octohedron as a dual of the cube. In other words, blob theory is all about fusion of instances into masses, and if you are in a mass there is no escape from it, there is only immanence, no transcendence. N-blob theory talks about the meta-levels of immanence. N-multiblob theory talks about the instances being faceted in such a way that they can be in solutions containing many mixed masses. To go with category theory is syllogistic logic. To go with blob theory is pervasion logic. These are two completely different kinds of logic. We can reason equally well in either system of logic. But since our tradition is obsessed with sets and transcendence it developed only a syllogistic logic and does not know the type of pervasion logic that was the norm in India or China traditionally. All dimensions have the simplex, the cross polytope and the tesseract. Thus we would claim that all dimensions support both intrapenetration and interpenetration, which is to say blob theory and category theory. This duality in geometry is a precursor or a hint at a deeper unrecognized duality in mathematics in general at the categorical level between blobs and categories. We don't like blobs and steer clear of them because they are ambiguous and ill defined. But they are unescapably there and our matheamatics would be healtier if we would recognize this fundamental duality from the start. Now it lies hidden from view and we can only see blob theory as through a glass darkly. But we need blob theory as the antidote to the sterility of transcendence without immanence. The reason mathematics does not relate to the world we live in very well is because of this exclusion of the other kinds of Being other than Pure Being, and especially the exclusion of Wild Being where Blob Theory has its natural home. Now if we take this view that the cross and tesseract polytopes are the intimation with regard to systems theory that there are two utterly different and dual ways for two systems to interact. And that this needs to be reflected at the most basic level of mathematics, then there is an interesting fact about the platonic solids, which is that there are five in three dimensions, six in four dimensions and three in every other dimension. In other words the dimensions that we live in the third and the fourth are strange as dimensions go, they have added structure. Interestingly the group expresses the rotations of the icosa/dodaca-hedron which is A5 is also the group of the Pentahedron. Thus there is an intimate and interesting connection between the dynamics of the highest 3-space platonic solids and the simplest four space platonic solid. But here is something also that is strange, which is that the 16-cell which is the analogue of the octahedron in four space is made up of tetrahedrons, and the analog of the octahedron is the 24-cell that is the unique platonic solid in four space that has no analogue in any other space. So this special non-blocking property of the octahedron is transferred to that unique structure which is also an all space filling lattice that combines the cross and tesseract polytopes that are dual lattices in four space. There is a twising when you move from three space to four space. The pentahedral simplex twists off at an angle from the tetrahdron, and the tetrahedron becomes identified with the 16-cell while the cube remains identified with the tesseract. The cube/tesseract remains connected and thus orthogonality remains the central feature of all the spaces. But at the simplex there is a twist introduced between 3-space and 4-space that allows the penahedron to shift away from the octahedron so that the 16-cell becomes the dual of the tesseract and the 24 cell inherits the properties of the octahedron. This strange twist along with the fact that there are further dual Platonic solids in 3-space and 4-space make these lower dimensions so much more interesting than the higher dimensions. Of course it is also in three space that knots become possible, but in four space all knots fall apart without being untied. So this is a fundamental difference between the two dimensions. In two space there are no platonic solids as there are infinite regular figures. In one-space there is only a line. So there are gigantic leaps of possibilities in the first few dimensions. This is similar to our argument that the first few simplicies have a special relation to the hyper-complex algebras. The first few dimensions also might have a special relation to the hyper-complex algebras. The pattern of the algebras and their upper threshold seems to be repeated throughout mathematics. What we are saying here is that there is a message in the strangness of the first few dimensions that carries across to systems theory ala B. Fuller's analysis of platonic solids as systems, or the interaction of systems, and then ultimately to our understanding of the simplicies in the context of both n-category and n-blob theories. There is a whole missing way of looking at mathematics which sees the big picture. The big picture in this case is the intrinsic relation between the dualities between the octahedron/cube and the tetrahedron that B. Fuller explored shows us that when we blow this up to the level of sets and category theory that there is a whole missing universe of mathematics associated with masses and blob-theory.

This missing duality between sets and masses appears at the very basis of mathematics. But we get a hint of it at the level of the platonic solids. Since the simplicies are platonic solids we must expect that the other platonic solids are important as well, especially those of low dimension. When we take the platonic solids as a way of thinking about systems, as establishing the thresholds of the complexity of thought, then we begin to see that every little detail of their structural relations to each other are important and need to be interpreted in the context of the field of all their relations. When we transfer that same duality to the basis of mathematics then we suddenly see that blob theory is missing and we have a lopsided mathematics that only recognizes sets and categories as the basis of mathematics. It is hard for mathematicians to think they have missed something so basic by being myopic. But we have to admit that there is something fundamentally wrong with out mathematics which does not connect to the world. Everyone knows mathematics is divorced from reality in some fundamental way despite the fact that it has been used in so many physial theories. But once we see what is missing, that it is a matter of the difference between sets and masses then we can begin to put things right based on the duality and balance between sets and masses. And in that way we can begin to see how the simplicies are in the position of the non-duals between the duals of fusion and interpenetration of systems. We have noted that the simplicies are a fusion of the characteristics of the autopoietic, dissipative and reflexive special systems. Now we start to see that they are an image of what we have called the ipsity that appears in conglomerates. The Pascal Simplicies are the conglomerate structure that gives us all possible elements that we see as either particulars or instances depending on whether we emphasize masses or sets. Now that we have gone up from the simplicies to the n-categories or the n-blobs we have taken a decisive step forward in our pusuit of understanding the infrastructure of the general schemas theory. The Platonic solids are our key to understanding the relation between n-category and n-blob theory. There is a reflection in mathematics that reverberates through it which taken together allows us to intuit the real basis of mathematics which is not lopsided as our mathematics of today. The fused state of the octahedron has these special properties of non-resistant flow that shows up in the 24-cell polytope that is made up of 24 intrapenetrated octahedrons. As we move up from the tetrahedron to the pentahedron of 4-space there is a twist around the position of the cube/tesseract that causes the pentahedron to be displaced out away from the tetrahedron, so that the tetrahedron ends up as being the source of the cross polytope which has tetrahedral cells in all higher dimensions. Thus fusion is cut off from higher dimensions. Fusion instead is made the center of the fourth dimension in a unique polytope that is the 24-cell. Orthogonality of the cube/tesseract is the axis around which this twist occurs. And we must admit that orthogonality is the basis of most of our higher math. The 16-cell and the 8-cell tesseract are dual all space filling lattices that together define the 24-cell all space filling lattice. In some sense the 24-cell merges with the 16-cell and the 8-cell lattices. This merger is a very interesting phenomena. There are four kinds of systems: normal, dissipative, autopoietic and reflexive. These four kinds of systems together make up the emergent meta-system. If we think of the normal system as the tetrahedron as B. Fuller would have us do then we can try to understand how the other platonic solids relate to the special systems. We can relate the fusion of the octahedron to the dissipative special system. Two systems when fused give the dissipative special system, where as if they are interpenetrated then they give the orthogonality of the greimas cube which is an expression of their interpenetration. The image of the autopoietic system appears as the relation between the icosa/dodaca-hedron and the pentahedron as it relates to the vector equlibrium archemedian solid. The 24-cell polytope represents the reflexive special system. Notice what happens here. The Vector equlibrium (VE) is formed by the close packing of spheres. It is an Archimedean solid made up of squares and triangles.

Each sphere is a model of a field of energy in which all forces are in equilibrium and whose vectors, consequently, are identical in length. When 12 spheres are packed closely around a central sphere, the resulting structure constitutes a polyhedron with 14 faces, namely 6 squares and 8 triangles. The centres of the surrounding spheres are the 12 vertices of what Fuller called the vector equilibrium.[4]

This VE, also called the Cuboctahedron, is the balance point of close packed spheres. It thus has a certain stability and it is also a midpoint between the inter and intra penetration of the octahedron and the cube. The VE is the epitome of the structural level of the autopoietic system. But this is balanced by the organizational level which we see in the relation between the icos-dodaca-hedron and the pentahedron of 4-space through the group A5. Notice that as we find the center between the fusion and interpenetration of the dual systems then we also move outside the normal progession by positing another source beyond the pentahedron in the same way we move from the tetrahedron to the penetahedron in the progression of the simplicies. So the VE is between the cube and octahedron, but the Pentahedron is beyond the tetrahedron and the cube opposite the octahedron and 24-cell. In this way the twist between the third and fourth dimension is precisely what produces the structure of the autopoietic system with its two layers. At one layer we see that the dissipative system is merely the fusion of two normal systems. But it has an opposite which is the orthogonal interpenetration of those systems instead. Then we posit that the next level because it is non-dual must be between and beyond this duality. So it is between in the sense that the VE is between the octahedron and the cube as a balance point of forces. But it is beyond in the sense that it is both the icosa/dodaca-hedron and the pentahedron in as much as they have the same group which is A5. Then once the twist has occurred that produces the autopoietic special system then we can project the opposite of the pentahedron as the special polytope of the 24cell that is the embodiment of the reflexive special system. Beyond that again is the meta-system of the 120/600 cell duality that are the other platonic solids in four space. In this way all the platonic solids are accounted for in our model of how the four kinds of systems create the meta-system. This model gives us all kinds of new information about the relations between the normal, special and meta-systems. It gives special meaning to the twist in the unfolding of the three to four

[4] http://www.nous.org.uk/VE.html

dimensional Platonic solids. And this new model of the Special Systems motivates us on our journey to understand the General Schemas Theory because again it has touched on the nature of the special systems and how where ever we look we find models of these special systems once we know what we are looking for. B. Fuller placed great emphasis on the VE. He recognized how important it was as an alternative to the octahedron and the cube. But B. Fuller did not go into the Four Dimensional Platonic Solids and explore them in the same way he explored the three dimensional platonic solids. Suddenly the Platonic solids of the third and fourth dimensions take on a new meaning. We see that based on B. Fullers identification of the system with the tetrahedron, we can go on to identify the octahedron as the model of the dissipative system, focusing on its special property of non-obstruction of flows. The tetrahedron can be combined with itself in two ways. One is fusion and the other is interpenetration. These duals define the possibility of a middle between them which is the vector equlibrium which has a unique stability. But this is just the structural level of the autopoietic system. For the organizational level we need to notice how the group A5 unites the icosahedron with all of its interesting properties with the pentahedron which has equally interesting properties. This introduces a twist because the duals produced in the fourth dimension are the 16-cell cross polytope and the 8-cell tesseract. The 16-cell is made up of tetrahedral, so it is related back down to the orginal tetrahedron. A new polytope at this level takes the place of the VE which is the 24 cell which is the model of the relflexive special system. The autopoietic special system is related to the VE and the pentahedron as well as the icosa-dodacahedron. The VE is the balance point between the 3-D duals. The icosa/dodacahedron relates to the boundary of the autopoietic system. The pentahedron relates to the hyper cycle that controls the autopoietic system. The twist between dimensions places the hyper cycle outside the system. What is above the system is the 16-cell and its dual the 8-cell. This allows a new non-dual to be projected which is the 24-cell. And then from there we can move up to the model of the meta-system which is the 120/600 cell. All this is completed before we move out of four dimensional space into the fifth dimension which like all higher dimensions is much less complex than the lower dimensions in terms of the number of different Platonic Solids. The line, triangle, tetrahedron, pentahedron are all simplicies and we go on up to the sextahedron in five dimensional space. The added structure in the third and fourth dimension with the other Platonic solids is what is necessary to specify the relations between the special systems geometrically. Octahedron and 24-cell share the non-blocking movement of directional lines within them as a special property. The pentahedron has the special property of being a fusion of two mobius strips and thus is an analog to the Kleinian bottle. The normal system is a simplex as is the control center of the autopoietic system. The control center of the autopoietic system shares the same group as the icosa/dodaca-hedron which represents its boundary. And the difference between the organizational level and the structural level is seen in the relation between the VE and the pentahedron. Once we have a simplex at the 4-D level then there is a projection of the 16-cell and the 8-Cell as further duals that are all space filling lattices. And the combination of these gives us the 24 cell lattice which has octahedral cells. It is a reflexive conglomerate produced out of dissipative fusions. From the 24 cell we can then project the 120/600 cell as the outer limit of the Platonic Solids which is a new boundary related to the meta-system. It is another complementary duality projected beyond the 16/8-cell duality. From this we learn that the dissipative and reflexive special systems share the ultra-efficacious fluidity of the octahedron and the 24-cell. On the other hand we also realize that the autopoietic system itself has three parts, which is a new realization. It has the hyper-cycle control structure of the pentahedron related to the 5-Hsing of Accupuncture, it has the boundary seen in the Icosa/Dodaca-hedron which shares a the same group structure A5 with between boundary and core. It also has a basis in the vector equlibrium which is non-dual between the octahedron and the cube. The VE is ultra stable. The boundary mediates between the VE and the pentahedron which is the same as saying that the boundary mediates between the structural and the organizational level which is a new way of looking at these relations. Within the autopoietic system there is again three components. There is the perfect balance of the VE. There is the Boundary between the VE and the pentahedron. And there is the Pentahedral hypercycle which is the intertwining of two mobius strips as a kleinian bottle. We have previously said that the Kleinian bottle is a model of the Autopoietic system topologically. Notice that the icosa/dodaca-hedron is dual and the pentahedron/VE is dual. So there are four pieces two of which are self dual and the other two of which are dual. This means that there are two faces to the boundary of the autopoietic system. These four geometric figures form a system between them. Three of these are in three dimensional space and one is in four dimensional space. The key here seems to be the group A5 which is the limit that prevents the solution of equations greater than degree 5. In those equations the variables cannot be rotated out and thus we cannot get a solution except by analysis. This fundamental blockage between the fourth dimension and the third dimension is an insuperable barrier. It is this insuperability of the barrier that throws us suddenly into thinking about immanence. This is what brings us back to the idea of n-blob theory as the dual of n-category theory. N-blob theory is the representation of not being able to move or transcend which appears in the relation of the pentahedron to the icosa-dodaca-hedron under the auspices of A5. Immanence is intrinsic. Just as intrinsic as Transcendence. But out culture tends only to develop the dualisms and forget the non-duals. Blob theory teases out this intrinsic immanance that appears suddenly out of no where between the third and fourth dimensions. N-blob theory talks about the meta-levels of masses. And n-multiblob theory talks about the solutions that are combinations of masses that contain instances that are faceted and thus relate to multiple masses at the same time. This is why our lowest General Schema is the facet. Instances in solutions of multiple simultaneous masses are faceted monads.

Let us go on to mention here the thirteen archemedian solids. These solids are regular but made up of different faces like the vector equlibrium. There are twelve archemedian solids other than the VE or Cuboctahedron. These twelve are manifestations of the basic twelveness that is playing itself out in the platonic solids as well. Notice this appears in the 120/600 cell. They are another image of the Meta-system where the various shapes are allowed to mingle rather than being separated as they are in the Platonic solids. This bakers dozen appears in various forms in the Western Tradition, as in the structure of the Mahabharata and the Greek Epics. The arkamedian solids are a face of the meta-system which is different from that of the 120/600 cell polytopes of 4d space. But they are so many forms that display possible complementarities. Not just the dual complementarity of the 120/600-cell but as a field of possibilities. We must keep our eyes open for the shapes of these fields and what they have to say to us about the nature of the meta-system and how it articulates itself in archetypal forms.

Now that we have established how the Platonic solids give some hints as to the nature of the importance of immanance. It is necessary to sketch out blob theory as it appears as a dual of category theory and the various meta-levels such as n-blob theory and n-multiblob theory. In this way we will be tracing out a type of mathematics that is the dual of that developed by Tom Leinster. When we look at the blobs we are considering how they encompass points. But when we look at the boundaries between the blobs we are concerned with topologies where non-orientable surfaces are representations of the special systems. As we have seen the VE tells us about the stability of close packed spheres. The Icosa/Dodaca-hedron tells us about the three dimensional boundary. The pentahedron tells us about the hyper cycle control structure in the imaginary realm beyond the structural that is the source of organization in the autopoietic system. Similarly here we are interested in the instances which are encompassed like the close packed spheres, we are interested in the boundaries that become topological surfaces. We note that in four dimensional space these cannot be classified because of Donaldson Fake R4 surfaces that are infinite. Finally we have to be interested also in the pentahedron, i.e. the places where the non-orientable surfaces connect to form kleinian bottles and other formations like the hyper-kleinian bottles. Boundary conditions get strange where non-orientable surfaces are concerned. All of these perspectives are important in Blob Theory. Blob Theory explores the immanence that holds all instances in place within their masses. But just because they are held in place does not mean that interesting things don’t happen to them there.

N-Blob Theory and N-MultiBlob Theory

Now it is time to look into the idea of Blob theory more deeply. We have produced an analogy out of the most basic mathematics, the Platonic Solids for the duality between Sets and Masses on the one hand and Categories and Blobs on the other. Now we want to extend Blob Theory in a way similar to the way Category theory has been extended into N-Category Theory[5] and N-MultiCategory Theory[6]. We note that the zero-category is considered the set, so the zero-blob is considered the mass. We have already noted previously that the mass and set each have their own logics accompanying them. Sets are reasoned about using Syllogistic Logic while Masses are reasoned about using Pervasion Logic in India and China. We have no equivalent for pervasion logic in the West. Syllogistic logics relate universals to attributes to substances. In Syllogistic logic we get to the conclusion by moving from finite exemplar, to universal, to attribute back to exemplar. So the universal is used as a bridge between the attribute and the exemplar. The classic syllogism, Socrates is a Man, All men are Mortal, therefore

[5] John Baez

[6] Tom Leinster

Socrates is Mortal shows this in action. First we find the Universal that Socrates is attached to, i.e. Man, then the attribute of that Universal, then we apply it back to Socrates the Exemplar. As we know there are three types of logical statements, Deduction, Induction and Abduction. Peirce identified Abduction as the other permutation of the three statements in the syllogism. Induction goes from particular to universal and Deduction as in the classic example goes from the Universal to the particular. But abduction produces a hypothesis and then seeks to connect that hypothesis to the universal and particular. In abduction the attribute is the key, in deduction the universal is the key, and in induction the exemplar is the key. This is syllogistic reasoning that is related to sets because sets are made up of a collection of unique different things. These things are exemplars that differ by their attributes. A set cannot have any two of the same item. Thus the set emphasizes the difference between particulars with specific different attributes. By collecting things in sets we are able to identify different kinds of things and thus see the universals. Sets by emphasizing difference between kinds of things helps us identify universals.

Masses are completely the opposite of Sets. Masses emphasize sameness between instances that make up the mass. The whole emphasis is on the emergent qualities of the mass itself over and above all the instances that make it up which are seen to be identical. Thus pervasion reasoning has to do with boundaries, not universals and whether some instance is within the boundary of the mass. We assume that there are three statements corresponding to the permutations of the syllogism about the relation of the mass to the boundary to the instance called devasion, invasion, and abvasion for simplicities sake. The boundary works like the universal. So devasion goes from the mass through the boundary to the instance. Invasion goes from the instance through the boundary to the mass. Abvasion goes from the boundary to both the mass and the instance. Each element has its point of departure and its endpoint from the three possible elements. Reasoning connects the point of departure to the end point through the mediating element. Notice that this is a triangle, and triangles have three points and three sides. Their lattice is 1331. This is then the simpliest of the pascal simplicies. The only thing simpler is the line and the point. It is the first simplex with any structure at all. But what this tells us is that there are probably logics associated with the higher simplicies as well, the tetrahedral simplex and the pentahedral simplex have their own more complex logic. We are still trying to comprehend the syllogism and pervasion logics. But if we look to the next level we see that it entails the Greimas Square of contraries and contradictions between universals and particulars that form a tetrahedron (AEIO).

If we allow ourselves to consider Mass-like pervasion logic as the dual of syllogistic logic, then the whole question for such a logic is whether the instance is within the boundary, and if it is then it participates in the mass. But we can get to this by either starting from the mass and looking for the instance via the boundary, or we can start at the instance then find the boundary and then see what mass it is part of. Or finally we can draw a boundary and construct the instances and mass that goes with that boundary. Let us consider for a moment the relation of the boundary to the universal. You see the universal takes the place of the mass for the particulars of the set. Similarly the boundary takes the place of the set for the instances of the mass. These are two complementary dual constructs. When we reverse the arrows of SET we get anti-SET, but this is not the MASS or the anti-MASS. Getting to the MASS we have to get to the bi-category level and see the functors between MASS and SET. There are four different possible bi-category permutations and these are what control the relations between Mass and Set categories. The whole of Mathematics is lopsided because it only recognizes Set as a fundamental category and has ignored Mass as a mathematical category just as basic. However, if we accept Mass as a Category we soon recognize that there is a problem because it is a very unusual category. In fact, we have to modify our concept of a category when we accept the mass into that same ilk with the set. This is where the possibility of Blob Theory arises. The Blob is the dual of the Category that is necessitated when we recognize the Mass is just as fundamental as the Set. It reorganizes all of math in a fundamental way if you accept Mass and Pervasion logic as part of mathematics. Categories are about morphisms between elements. In fact all the properties of the elements are determined by the morphisms in Category Theory. Thus you really don't need the elements any more. So category theory jettisons the elements and keeps the arrows. With the arrows you can describe everything about the category and all its elements because the morphisms contain are what confers on the elements their differences from the point of view of Category Theory. This is a very interesting development. This is to say that the differences are explained by the differences between associative arrows at the 1-category level. Now if we were going to do the dual of this for Masses then we would say that the similarities were explained by boundaries at the 1-Blob level. In other words the 1-Blob boundaries are simple envelopes or distinctions that separate those instances within and without of the Mass. Similarity means being within the mass and dissimilarity means being outside the mass. The mass is then characterized by its boundary more than anything else and sameness of the instances within is determined by the relation of any instance to the boundary. There is no transcendental relation between instances as there are between particulars. All instances within a Mass boundary are immanent only and determined by the mass they are a part of. So when we go up to the 2-category level we begin to see functors between categories. Functors are morphisms between two categories that allow us to prove something in one category and then transfer it to structures in another category by recognizing parallels. Note the 2-Blob is when a boundary encompasses two masses. But this also sets up a tissue between the two masses that is an interface and a barrier. When this becomes interesting is when we are dealing with an ipsity of a conglomerate and the ipsity is treated as both an instance and a particular at the same time. You see Masses have attributes not instances. Particulars have attributes not sets. So in fact these two approaches are turned upside down in relation with each other. It is the particulars that are like the mass and it is the instances that are like the set. So when we chain these together we can see ourselves going from instance to mass that is then seen as a particular which is part of a set that is then an instance of something higher. Or we can refuse the inversion and allow the contradiction that something is both a particular and an instance at the same time. Either way we get strange combinations of set and mass characteristics for their instances and particulars once we get to the 2-category/2-blob level. Because we can allow the functors to cross over into the mass and allow the boundaries to cross over into the set. This of course leads to a lot of degenerate cases. We wont follow that line of construction here because that would be a diversion. Needless to say the complementarity between the 2-category and the 2-blob is more difficult to see, especially in the mixed case. But now there are enough elements at play that there are many permutations that are possible in constructing various combinations of blob and category at the second meta-level. And as we go on up the hierarchy of meta-levels things get more difficult at each stage. The 3-category is the natural transformation in which one kind of thing turns into another kind of thing. The equivalent to this is the 3-blob level which we called a bag. A bag collects a lot of different tissues. So while the kinds of categories are transforming into each other the masses are consolidating and their tissues are being collected into a bag which is a kind of meta-meta-boundary of masses of masses. Then on the Category side we go to the 4-category which is called a modification and we have countered that with the term tweak. It is difficult to say what a meta-meta-meta boundary might be. I assume on the category side it means that if we transform between kinds then there are small modifications, like mutations, or crossovers that determine the differences between categorical kinds. Similarly when we collect the masses in a bag there might be tweaks that determine the differences between bags. So we get statements like "Yes Sir. Yes Sir. Three bags full." Where each bag contains something slightly different. Tweaks determine those slight differences. But all this is just reasoning from the parallelism between n-categories and n-blobs. What we need to do is to work out what n-blobs are on their own and then compare them back to the n-categories to make sure that a robust duality continues to exist at each meta-level. But this means being more specific about what n-blob theory is like at a formal level and actually considering more deeply what it means to move to meta-levels on the blob side rather than the category side. Baez's Opetope is the picture of what it is like to move up the ladder of meta-levels on the n-category side. What we need is something like the Opetope as a means of reasoning about the higher levels of n-blob theory. And at this point we are not there yet. Opetopes are an analogy between operators and polytopes and allow gluing operations. Presumably we need some other metaphor than gluing, some sort of winnowing perhaps that separates out instances using different filters so that it can be deterimined which masses that they belong to. Mass operators are like the arrows, they are like waves that disturb the instances of the mass of the sea. But with regard to instances there are no arrows, but only their immanence within the mass that can allow them to be filtered and thus winnowed out as to what masses they belong to. Without a clear view as to the nature of this dual metaphor it is difficult to see how to develop n-blob theory as a separate entity from n-category theory yet one that has its own integrity and that remains dual in every respect throughout the process of creating the n-blob levels. Also we must consider solutions, which are the n-multiblobs that are the equivalent to the n-multicategories of Leinster. Solutions are multiple masses at the same time that instances participate in simultaneously. Where arrows are multi-input and one output, solutions are multi-output from one instance. This is why ipsities must be faceted in general schemas theory. The facet is the lowest level schema because it interfaces with this necessity of representing multiple masses as a solution pervading a single instance. So where n-multicategories fan in we see that n-multi-blobs fan out and this is another way they complement each other. The n-multicategory theory naturally produces algebras. The question is what does n-multiblob theory produce? Probably qualities through the interpenetration of masses in Venn diagram like representations that are 2^N representations. Algebras are quantitative and are given over to N^2 type representations. Interactions tend to appear as vector and matrix operations in Grassman like algebras. In all these orthogonality is the key to calculation. But on the other side we have fusion which denies the maintenance of orthogonality. So this is where we must start to be careful for how do we describe accurately this fusion. We are starting to leave math behind and need qualitative tools like the heuristic models that the Chinese developed using Ying and Yang and various permutations of those variables for opposites.

The thing to notice about Baez's opetopes is that they are using the simplices as the basis for understanding how higher order operators form polytope like connections with each other. It is using the fact that the Pascal Triangle is describing these higher order combinations of operators which become the Pascal Simplicies in order to construe the ways of gluing together the operators to get higher order operators. Now we can make a similar appeal with regard to the n-blob theory. We can think of each layer of the Pascal Triangle in relation to the whole of the Triangle as the relation mass to instance, and again of each element in the layer to the whole layer as mass to instance. In other words the fractal like structure of the Pascal Triangle lends itself to a Mass like description and perhaps we can understand this description in a way that gives concrete meaning to the n-blob theory we are trying to construct.

First we note that the Pascal Point is "1". The pascal lines are a series of "1111…" and when they cross we get an interference pattern on in one hourglass and nothing in the other hourglass. Now as the Pascal Triangle dissipates its order outward it forms a mass of instances. Each layer has its own structure that is defined as a group of 2^n elements. For instance there are groupings 1.4.6.4.1 of sixteen elements at 2^4. Each element is a patterning of four bits. At the next layer there are groupings of 1.5.10.10.5.1 of thirty two elements at 2^5. Each element is a patterning of five bits, and so on. Now each element is an instance which is built up by a permutation of four or five opposites in the two layers just mentioned. These instances have no properties of their own, they are merely differentiated in some minimal fashion. However, even that minimal differentiation has a structure that is seen in the groupings. These groupings occur as we take out substitution and inversion, i.e. the mirroring which is ramifying as the Pascal Triangle grows. So let us consider first the mass to be the layer and the instance to be the bit pattern that differentiates the elements. We notice that there are boundaries around the layer which are the two singular elements at the extremes of the interval which we see as the ones of the crossing Pascal lines. But there are also the boundaries which are two sided which separate the elements from each other into groups of elements. So in layer production we are getting a mass effect because the whole layer has a particular structure unique to the mass as a whole. Instances only have the necessary differentiation for them to be recognized as different from each other individually, but this minimal differentiation still produces a mass like emergent structuring that groups the elements in a particular layer. Now when these layers are recognized to be simplex polytopes then we can talk about each one having particular properties that make it unique in its dimensionality because of the assignment of points, lines, faces, solids, hyper-solids, etc. Notice that points, lines, faces, solids, hypersolids, etc are set like qualities that we project on the numbers in the Pascal triangle to get to this geometrical interpreation. The geometrical interpretation that leads to the idea of the opetope is dependent on a set-like approach in as much as each simplex is of a unique kind. But if we refrain from this set-like approach, then and instead focusing on the differentiation of the minimal elements as instances in mass layers then we get a completely different view of the simplicies which is mass-like rather than set-like. So there are two boundaries to the layer, one is the limiting elements, all 1 bits or all 0 bits, and the other is the grouping of elements that reflects the mirroring symmetries. When we move up a level and take another vista we can see the whole of the Pascal Triangle as a Mass. In that case the elements are layers 2^n and the boundaries are between these layers. The limits of this higher level interval are odd zero, void, and positive infinity. This higher order mass is expanding as a dissipative order while the lower level mass of the layer is static as a step in that expansion. Now if we understand the instances, as element or layer, of the two masses, layer or triangle as a whole, then the question becomes what is the nature of the 2-blob boundary in relation to the 0-Blob and the 1-Blob boundaries. The 0-Blob has to be the instances. We can think of the 1-Blob boundaries as the limits, in each case. This is equivalent to the morphemes or arrows in Category theory. Morphisms cross the gulf between elements to produce the structure of the category and give the elements their properties. So the 2-blob boundaries would be the internal differentiation within the layer or between layers in terms of the whole Pascal Triangle. We have called these 2-blob boundaries tissues and this seems to be an appropriate name. They are part of the internal differentiation of the internal structure of the simplicies. Functors are arrows or morphisms between categories. But tissues are boundaries within the exernal boundaries of the mass. So here we get a radical difference between inner and outer. Functors are outer 2-differences while tissues are inner 2-differences. This is where we begin to see the radical difference between Blob Theory and Category Theory. In Blob Theory you cannot go outside to reach any point beyond the limit of the mass, but you can go inside to realize the fine structure of the differentiation between the various kinds cells within the mass. Now if the 2-Blob boundaries are the internal discontinuities that are structural, then the next question is what are the 3-Blob boundaries. I would say that they are the differentation between the element-layer and the layer-triangle structures. We have called this differentation a bag and it corresponds to the natural transformations in category theory. Natural Transformations turn one kind of category into another kind of category because they are 3-arrows between functors. But in Blob Theory we have instead the realization that the various levels of the Blob are mirrorings of each other. In other words Layers and the Triangle as a whole are encompassed by the same bag, which in this case is the limit of the two Pascal Lines the interference patter of which is the Pascal Triangle. So where 2-Blob tissues were internal 3-blob bags are the realization of the internal coherence of the whole mass due it its mirroring of itself on different levels. Natural Transformations turn one kind of category into another kind of category. This is a transformation that causes the essence of the category to change, that is a transformation of the deepest kind. On the other hand the Bag boundary really says that the boundary of the layer and the boundary of the triangle as a whole is the same, which is true because it is the same two Pascal lines that are represented as all 1 bits or all 0 bits. The Bag boundary reinforces the boundary of the mass. It has the opposite effect of the natural transformation in that it makes the mass more the same rather than different from itself as are categories undergoing natural transformations. We can say that 2-blob boundaries are fine structure of the mass while 3-blob boundaries establish external coherence of the Mass. Next we must ask what the meaning of the 4-blob boundaries might be which we have called tweaks as opposed to the modifications of 4-arrows of category theory. If in category theory we have transformed one kind of category into another then the next level up cannot make much of a change beyond that radical change. So this small change that is left as a possibility is called a modification. Notice that this series of steps is like the series that Bateson Talks about with respect to learning and physics. In physics there is stillness(0), then motion(1), then acceleration(2), then acceleration of acceleration(3) and then jitters(4). Jitters is the fourth meta-level of motion. These Blob and Categorical meta-levels are much the same. Stillness is like the instance or particular at the zero level of Blob or Category theory. Motion at the first level is like the transcendence of the morphic arrow or like the establishment of a boundary or distinction. Acceleration at the second level is like the functors between categories or the establishment of the tissues of internal differentiation. Acceleration of acceleration at the third level is like the natural transformation that changes kinds or like the bag which reinforces internal coherence. Jitters at the fourth level is like the modifications or tweaks. Not much room is left for change. With respect to changes in kinds we can only make minor modifications. With respect to the reinforced internal coherence of the bags we can only make small tweaks. Of course these levels correspond to the kinds of Being. As Bateson says it is very difficult to think any motion beyond the fourth meta-level just as it is difficult to think of any kind of learning beyond the fourth meta-level. So in terms of both physus and logos the fourth meta-level seems to be the horizon of the thinkable. So we have a paradox which is that although there is an infinite sequence of n-blob or n-category meta-levels it seems impossible to think beyond the fourth meta-level of these sequences. Yet they exist, unthinkable as they are. We can reach out into that existence beyond thinkability using the method of opetope theory, i.e. thinking up the layers of the simplex as geometrical representations of operations. Similarly we can think up the levels of existence within the masses as well, because the simplexes go on. The Pascal Triangle is just the beginning. It is the oracle that sets out the pattern for all higher simplicies to follow. We can see these higher simplicies as n-dimensional masses. The reations between these masses and the lower level masses allow us to map out the relations between the various n-blob boundaries because at each level a new kind of boundary is needed to differentiate this lower level mass from the new higher level mass that has now appeared as the simplicies unfold. But let us go back and consider the modification and the tweak which exist at the level of Wild Being. I would say that in the case of the tweak what we have is the realization of the negative pascal triangle that balances the positive one. The tweak is what pushes us over the edge of the boundary of the void into the negative dimensionality. Once we have been pushed over that boundary then we see the relation between models of interpenetration in the negative dimension and the positive layers themselves. Each layer in the triangle has its negative dimensional dopple ganger. The dopple ganger is a model of interpenetration based on the hyper-complex algebras. Thus the external differences between elements is balanced by their interpenetration in the negative dimensionality. So this tweak which throws us unexpectedly into the negative dimensional double of the Pascal Triangle is a very significant one. This is far different from the modification of kinds that appears in category theory. Modifications of kinds give us dualities like the difference between male and female of the kind human. These are very important external differentiations between modifications of a kind. But in N-blob theory the tweak again increases internal coherence instead of producing external differentiation. At every level of the unfolding of the simplicies the same set of n-Blob boundaries appear over and over again. But each unfolding of the simplicies is more complex so that by implication a new higher level n-boundary is created. So when we go up to the Pascal Tetrahedron it is only necessary to go to the next reference level of the Pascal Triangle which is 1.4.6.4.1. We see that reference level unfold into a triangle which has this same pattern along each side and three12s in the middle.

```
1       1     1        1          1
       1 1   2 2      3 3        4  4
           1 2 1     3 6 3      6 12 6
                   1 3 3 1    4 12 12 4
                               1 4  6  4 1
```

Then once the reference layer has been constructed then one constructs each layer back to unity and this is the form of the simplex in that dimension, in this case the tetrahedron. But if one turns toward infinity then there are infinite layers from the reference layer beyond the simplex proper. Along each edge of the tetrahedron is a Pascal Line of 1s. Each face is a Pascal Triangle which is independent of the other two triangles on the other faces. The triangle along the edge that is expanding is not a Pascal Triangle but is composed of the interior points that are generated by adding the three nearest neighbors from the last layer. Now the meaning of the layer has changed to 3^n. Bits now have three states instead of two. But essentially we see that there are still elements which are grouped to form a layer, and the layer is a step in the progression of the dissipative order away from unity of the whole tetrahedral mass. Again there are tissues which are represented by the blank spaces in the simplex inverse lattice. The bag is the reinforced coherence between the tetrahedron as a whole and its layers. And the tweak would uncover the negative dimensional tetrahedron which allows all the elements to interpenetrate. Yet there is also the boundary which is dimensional between the tetrahedron and the triangle as such. We might consider this the 5-blob boundary. It extends the mirroring that was happening in the Pascal Triangle now to three dimensions. This new mirroring introduces a deepening coherence to the Pascal Tetrahedron missing from the Pascal Triangle. So each mass at each new dimensional level becomes more coherent and that increased coherence due to additional mirroring is what makes these models increasingly reflexive. The new boundary in this case is the axis of the tetrahedron from unity which goes down the middle of the tetrahedron. Each dimension will produce a different axis from a different source of unity in that new dimension. In each case the new axis is a boundary that confers greater internal coherence though mirroring. So these axes can be seen as the n-blob boundaries of each higher simplex. Each one is deeper inside. At each stage there is increased coherence. Each simplex conjunctively contains all the previous boundaries. We can start over counting the n-blob boundaries at each simplex and we will always find elements n^n within layers that appear in groups. We will always find that layers are instances within the overall new simplex. Thus the normal limiting boundaries of the Pascal Lines always exist as the equivalent to the morphism. The inside differentiations which appear as the empty cells will always be there as tissues creating internal differentiation. The layer and the whole simplex will always be contained in the same Bag. The tweak that throws us into the negative dimensionality will always uncover models of interpenetration in negative dimensions. But then there is always the new axis of mirroring at the core of the new mass which can be seen as the n-blob boundary associated with that mass.

So we have seen that it is possible to create an opetope like strategy to understand the higher meta-levels of n-blob theory just as Baez has done for n-category theory. Interestingly this strategy forces us in both cases back to the simplex as a model both for masses and for sets. This increases the coherence between n-blob theory, n-category theory and the theory of Pascal simplicies. Suddenly we see how the transform between masses and sets appear when they are projected upon the simplex. These strategies are born from desperation because we have no examples to work from of the higher categories or the higher blobs. Category theory was born out of the analysis of the similarities and differences of known mathematical categories. But in the case of n-category theory we do not have those examples to work from and we must instead invent them. This invention is necessary both on the side of n-category theory and n-blob theory. Both therefore triangulate back to the simplex that is the only independent element left. By seeing both n-blob theory and n-category theory triangulate back we get a better picture of their complementarity.